

Heat Kernel Expansion and Extremal Kerr-Newmann Black Hole Entropy in Einstein-Maxwell Theory

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Abstract

We compute the second Seely-DeWitt coefficient of the kinetic operator of the metric and gauge fields in Einstein-Maxwell theory in an arbitrary background field configuration. We then use this result to compute the logarithmic correction to the entropy of an extremal Kerr-Newmann black hole.

1 Introduction

In a classical two derivative theory of gravity, the Bekenstein-Hawking formula gives an expression for the entropy of the black hole. This formula is universal in the sense that it does not depend on the details of the matter field content of the theory. Furthermore it does not depend on any specific ultraviolet completion of the theory – an independent computation of the black hole entropy from counting of microstates must reproduce this result in any consistent quantum theory of gravity. This provides a strong constraint on consistent ultraviolet completions of the theory.

The Bekenstein-Hawking result for the entropy is expected to receive quantum corrections which are subdominant for black holes of large size but could become important for black holes of size comparable to Planck size. The details of these corrections will depend on the ultraviolet completion of the theory, but there is a certain class of corrections, proportional to the logarithm of the size of the black hole, which receive contribution from loops of massless fields propagating in the black hole background and can be computed purely from the knowledge of the infrared physics [1–12]. In particular for a class of extremal black holes in four dimensional $\mathcal{N} = 4$ and $\mathcal{N} = 8$ supersymmetric string theories and five dimensional BMPV black holes [13, 14] the logarithmic corrections to the black hole entropy computed from infrared physics correctly reproduce the result from the microscopic counting in string theory [6, 7, 9].

Since the procedure used in [6–9] for computing logarithmic correction to the entropy does not rely on supersymmetry, this can be carried out for any extremal black hole. This requires computing the heat kernel (more precisely its expansion coefficient a_4 introduced in eq.(2.5)) of the kinetic operator of the massless fields in the black hole background. Keeping this motivation in view we study the short distance expansion of the heat kernel in Maxwell-Einstein theory for fluctuations of gauge and graviton fields around an arbitrary background field configuration and compute the relevant quantity a_4 that appears as the coefficient of the logarithmic correction to the black hole entropy. The result is given in eq.(2.17). We then evaluate this in the background of an extremal Kerr-Newmann black hole solution and find an explicit expression for the coefficient of the logarithmic correction to the black hole entropy. The result is in eq.(3.17). Any microscopic explanation of the entropy of such a black hole, *e.g.* via Kerr-CFT correspondence [15, 16], should reproduce the result given in (3.17).

2 Heat kernel expansion in Einstein-Maxwell theory

We consider Euclidean continuation of Einstein-Maxwell theory in four space-time dimensions with action

$$\mathcal{S} = \int d^4x \sqrt{\det g} \mathcal{L}, \quad \mathcal{L} = [R - F_{\mu\nu} F^{\mu\nu}] , \quad (2.1)$$

where R is the scalar curvature computed with the metric $g_{\mu\nu}$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the gauge field strength. Note that we have set $G_N = 1/16\pi$. Let $(\bar{g}_{\mu\nu}, \bar{A}_\mu)$ denote any solution to the classical equations of motion in this theory and let $\bar{F}_{\mu\nu} \equiv \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu$ be the corresponding gauge field strength. We consider fluctuations around this background of the form

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{2} h_{\mu\nu}, \quad A_\mu = \bar{A}_\mu + \frac{1}{2} a_\mu , \quad (2.2)$$

and denote by $\{\phi_m\}$ the set of all the fluctuating fields $\{h_{\mu\nu}, a_\mu\}$. Then to quadratic order in the fluctuations the action takes the form:

$$\frac{1}{2} \int d^4x \sqrt{\det \bar{g}} M^{mp} \phi_p K_m^n \phi_n , \quad (2.3)$$

for some matrix M and a differential operator K of the form

$$K_m^n = (D^\mu D_\mu)_m^n + (N^\mu D_\mu)_m^n + P_m^n , \quad (2.4)$$

where D_μ denotes ordinary covariant derivative with connections determined by the background metric and N^μ and P are appropriate matrices constructed from the background fields. All indices are raised and lowered by the background metric $\bar{g}_{\mu\nu}$, and the total derivative terms are adjusted so that the operator K is hermitian. Then the heat kernel is defined to be the operator e^{sK} , and $\text{Tr } e^{sK}$ has a small s expansion of the form [17–25]:

$$\text{Tr } e^{sK} = \int d^4x \sqrt{\det \bar{g}} \sum_{n=0}^{\infty} s^{n-2} a_{2n}(x) , \quad (2.5)$$

where the coefficients $a_{2n}(x)$ – known as the Seely-DeWitt coefficients [17] – are expressed in terms of local invariants constructed from the background metric, Riemann tensor, gauge field strength and their covariant derivatives. Our goal will be to compute the coefficient a_4 for an arbitrary background $(\bar{g}_{\mu\nu}, \bar{A}_\mu)$ satisfying classical equations of motion, and then use this to compute logarithmic corrections to the entropy of an extremal Kerr-Newmann black hole using the results of [6–9].¹

¹On manifolds with boundary we can also have boundary terms with half integral powers of s .

Our strategy for computing a_4 is as follows. First we express the differential operator K_m^n as

$$K_m^n = (\mathcal{D}_\mu \mathcal{D}^\mu)_m^n + E_m^n, \quad (2.6)$$

where

$$\begin{aligned} \mathcal{D}_\mu &= D_\mu + \omega_\mu, \quad \omega_\mu = \frac{1}{2} \bar{g}_{\mu\nu} N^\nu, \\ E &= P - \bar{g}^{\mu\nu} (\omega_\mu \omega_\nu + D_\mu \omega_\nu). \end{aligned} \quad (2.7)$$

If

$$\Omega_{\mu\nu} \equiv [\mathcal{D}_\mu, \mathcal{D}_\nu], \quad (2.8)$$

denotes the curvature associated with the covariant derivative \mathcal{D}_μ , then we have [24]

$$\begin{aligned} a_4(x) &= \frac{1}{360 \times 16\pi^2} \text{tr} \left[60 D^\mu D_\mu E + 60 R E + 180 E^2 + 12 D_\mu D^\mu R \right. \\ &\quad \left. + 5 R^2 - 2 R_{\mu\nu} R^{\mu\nu} + 2 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + 30 \Omega_{\mu\nu} \Omega^{\mu\nu} \right]. \end{aligned} \quad (2.9)$$

Here $R_{\mu\nu\rho\sigma}$, $R_{\mu\nu}$ and R denote respectively the Riemann tensor, Ricci tensor and scalar curvature computed from the background metric and tr denotes trace over the index m labelling all the fluctuating fields ϕ_m . In our analysis we shall ignore total derivative terms which do not contribute to the integral in (2.5). Also for any classical solution we have $R = 0$ and hence we can ignore terms proportional to R .

We gauge fix the Einstein-Maxwell theory by adding a gauge fixing term

$$- \int d^4x \sqrt{\det \bar{g}} \left[g^{\rho\sigma} \left(D^\mu h_{\mu\rho} - \frac{1}{2} D_\rho h^\mu{}_\mu \right) \left(D^\nu h_{\nu\sigma} - \frac{1}{2} D_\sigma h^\nu{}_\nu \right) + \frac{1}{2} D^\mu a_\mu D^\nu a_\nu \right], \quad (2.10)$$

and add the corresponding ghost action. The total gauge fixed action up to quadratic order in the fluctuations is given by

$$\begin{aligned} \mathcal{S} &= \frac{1}{2} \int d^4x \sqrt{\det \bar{g}} \left[-h^{\mu\nu} (\Delta h)_{\mu\nu} + a_\mu \{ (D_\rho D^\rho)^{\mu\nu} - R^{\mu\nu} \} a_\nu \right. \\ &\quad - \frac{1}{2} \bar{F}_{\mu\nu} \bar{F}^{\mu\nu} \{ (h^\rho{}_\rho)^2 - 2h^{\rho\sigma} h_{\rho\sigma} \} - 4 \bar{F}_{\mu\nu} \bar{F}_{\rho\sigma} h^{\mu\rho} h^{\nu\sigma} - 8 \bar{F}_{\mu\rho} \bar{F}_\nu{}^\rho h^\mu{}_\sigma h^{\nu\sigma} + 4 \bar{F}_{\mu\rho} \bar{F}_\nu{}^\rho h^\sigma{}_\sigma h^{\mu\nu} \\ &\quad - \sqrt{2} \bar{F}^{\mu\nu} h^\sigma{}_\sigma f_{\mu\nu} + 4\sqrt{2} \bar{F}^{\mu\nu} h_\nu{}^\rho f_{\mu\rho} \\ &\quad \left. + 2b^\mu (\bar{g}_{\mu\nu} \square + R_{\mu\nu}) c^\nu + 2b \square c - 4b \bar{F}_{\mu\nu} D^\mu c^\nu \right], \end{aligned} \quad (2.11)$$

where

$$f_{\mu\nu} \equiv \partial_\mu a_\nu - \partial_\nu a_\mu, \quad (2.12)$$

$$\begin{aligned} (\Delta h)_{\mu\nu} = & -\square h_{\mu\nu} - R_{\mu\tau} h^\tau{}_\nu - R_{\nu\tau} h^\tau{}_\mu - 2R_{\mu\rho\nu\tau} h^{\rho\tau} + \frac{1}{2} \bar{g}_{\mu\nu} \bar{g}^{\rho\sigma} \square h_{\rho\sigma} \\ & + R h_{\mu\nu} + (\bar{g}_{\mu\nu} R^{\rho\sigma} + R_{\mu\nu} \bar{g}^{\rho\sigma}) h_{\rho\sigma} - \frac{1}{2} R \bar{g}_{\mu\nu} \bar{g}^{\rho\sigma} h_{\rho\sigma}. \end{aligned} \quad (2.13)$$

b_μ, c_ν are the diffeomorphism ghosts and b, c are the ghosts associated with the gauge invariance of the Maxwell action. Note that the ghosts will contribute to the trace in (2.9) with an overall negative sign.

We can determine the matrices M, N, P by comparing (2.11) with (2.3), (2.4) and then determine ω_μ, E and $\Omega_{\mu\nu}$ using (2.7), (2.8). This in turn allows us to compute a_4 via (2.9) in terms of the background fields. We can simplify the expression using the equations of motion:

$$D^\mu \bar{F}_{\mu\nu} = 0, \quad R_{\mu\nu} = 2\bar{F}_{\mu\rho} \bar{F}_\nu{}^\rho - \frac{1}{2} \bar{g}_{\mu\nu} \bar{F}_{\rho\sigma} \bar{F}^{\rho\sigma}, \quad (2.14)$$

and the Bianchi identities:

$$D_{[\mu} \bar{F}_{\nu\rho]} = 0, \quad R_{\mu[\nu\rho\sigma]} = 0. \quad (2.15)$$

The computation is tedious but straightforward and so we shall not give the details of the intermediate steps. Up to addition of total derivative terms the non-vanishing contribution to a_4 (including the contribution from the ghosts) comes from the following terms:

$$\begin{aligned} \text{tr}(-2R_{\mu\nu} R^{\mu\nu} + 2R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}) &= 4(-2R_{\mu\nu} R^{\mu\nu} + 2R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}) \\ \text{tr} E^2 &= 3R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 9R_{\mu\nu} R^{\mu\nu} + 3R_{\mu\nu\rho\sigma} \bar{F}^{\mu\nu} \bar{F}^{\rho\sigma} + 9(\bar{F}^{\mu\nu} \bar{F}_{\mu\nu})^2 \\ \text{tr}(\Omega_{\mu\nu} \Omega^{\mu\nu}) &= -5R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + 56R_{\mu\nu} R^{\mu\nu} - 18R_{\mu\nu\rho\sigma} \bar{F}^{\mu\nu} \bar{F}^{\rho\sigma} - 54(\bar{F}^{\mu\nu} \bar{F}_{\mu\nu})^2. \end{aligned} \quad (2.16)$$

Substituting these in (2.9) we get

$$a_4(x) = \frac{1}{360 \times 16\pi^2} (398R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + 52R_{\mu\nu} R^{\mu\nu}). \quad (2.17)$$

This will be one of our central results.

Before we proceed we note several points:

1. The terms proportional to $R_{\mu\nu\rho\sigma} \bar{F}^{\mu\nu} \bar{F}^{\rho\sigma}$ and $(\bar{F}^{\mu\nu} \bar{F}_{\mu\nu})^2$ cancel in the final expression. Since the final result is given only in terms of the background metric, the result is invariant under electric-magnetic duality rotation.

2. For the euclidean near horizon geometry of a Reissner-Nordstrom black hole:

$$ds^2 \equiv \bar{g}_{\mu\nu} dx^\mu dx^\nu = \Lambda^2 (d\eta^2 + \sinh^2 \eta d\theta^2) + \Lambda^2 (d\psi^2 + \sin^2 \psi d\phi^2), \quad \bar{F}_{\psi\phi} = i \Lambda \sin \psi, \quad (2.18)$$

eq.(2.17) gives $a_4 = 53/(90\pi^2 \Lambda^4)$. This agrees with the explicit computation of a_4 using the eigenvalues of the kinetic operator K [8].

3. Even though the final result is given in terms of the background metric only, we shall get wrong result if we ignore the terms involving $\bar{F}_{\mu\nu}$ from the beginning. Part of the contribution involving $R_{\mu\nu} R^{\mu\nu}$ term in (2.17) comes as a result of eliminating the combination $\bar{F}_{\mu\rho} \bar{F}_\nu{}^\rho$ in terms of $R_{\mu\nu}$ using (2.14).

3 Logarithmic correction to Kerr-Newmann black hole entropy

Next we shall use (2.17) to compute logarithmic correction to the entropy of an extremal Kerr-Newmann black hole. The metric of a general Kerr-Newmann black hole is given by

$$\begin{aligned} ds^2 &= -\frac{r^2 + a^2 \cos^2 \psi - 2Mr + Q^2}{r^2 + a^2 \cos^2 \psi} dt^2 + \frac{r^2 + a^2 \cos^2 \psi}{r^2 + a^2 - 2Mr + Q^2} dr^2 + (r^2 + a^2 \cos^2 \psi) d\psi^2 \\ &\quad + \frac{(r^2 + a^2 \cos^2 \psi)(r^2 + a^2) + (2Mr - Q^2)a^2 \sin^2 \psi}{r^2 + a^2 \cos^2 \psi} \sin^2 \psi d\phi^2 \\ &\quad + \frac{2(Q^2 - 2Mr)a}{r^2 + a^2 \cos^2 \psi} \sin^2 \psi dt d\phi \\ &= -\frac{(r^2 + a^2 \cos^2 \psi)(r^2 + a^2 - 2Mr + Q^2)}{(r^2 + a^2 \cos^2 \psi)(r^2 + a^2) + (2Mr - Q^2)a^2 \sin^2 \psi} dt^2 \\ &\quad + \frac{r^2 + a^2 \cos^2 \psi}{r^2 + a^2 - 2Mr + Q^2} dr^2 + (r^2 + a^2 \cos^2 \psi) d\psi^2 \\ &\quad + \frac{(r^2 + a^2 \cos^2 \psi)(r^2 + a^2) + (2Mr - Q^2)a^2 \sin^2 \psi}{r^2 + a^2 \cos^2 \psi} \sin^2 \psi \\ &\quad \times \left(d\phi + \frac{(Q^2 - 2Mr)a}{(r^2 + a^2 \cos^2 \psi)(r^2 + a^2) + (2Mr - Q^2)a^2 \sin^2 \psi} dt \right)^2, \end{aligned} \quad (3.1)$$

where for $G_N = 1/16\pi$ the physical mass m , charge q and the angular momentum J are related to the parameters M , Q and a via

$$m = 16\pi M, \quad J = 16\pi M a, \quad q = 8\pi Q. \quad (3.2)$$

The horizon is located at

$$r^2 - 2Mr + a^2 + Q^2 = 0, \quad (3.3)$$

and the classical Bekenstein-Hawking entropy, given by $1/4G_N = 4\pi$ times the area of the outer event horizon, takes the form

$$S_{BH} = 16\pi^2 \left[2M^2 - Q^2 + 2M\sqrt{M^2 - (a^2 + Q^2)} \right]. \quad (3.4)$$

The extremal limit corresponds to $M \rightarrow \sqrt{a^2 + Q^2}$. In this limit we get

$$S_{BH} = 16\pi^2(2a^2 + Q^2) = \sqrt{(q^2/4)^2 + (2\pi J)^2}. \quad (3.5)$$

To take extremal limit of the near horizon geometry we introduce a new parameter λ and new coordinates ρ, τ, χ via (see *e.g.* [26]):

$$M^2 = a^2 + Q^2 + \lambda^2, \quad r = M + \lambda\rho, \quad t = (2a^2 + Q^2)\tau/\lambda, \quad \phi = \chi + \frac{a}{Q^2 + 2a^2} \left(1 - \frac{2M\lambda}{Q^2 + 2a^2} \right) t, \quad (3.6)$$

and take the $\lambda \rightarrow 0$ limit keeping ρ, τ fixed. In this limit the metric (3.1) reduces to

$$\begin{aligned} ds^2 = & (Q^2 + a^2 + a^2 \cos^2 \psi) \left(-(\rho^2 - 1)d\tau^2 + \frac{d\rho^2}{(\rho^2 - 1)} + d\psi^2 \right) \\ & + \frac{(Q^2 + 2a^2)^2}{(Q^2 + a^2 + a^2 \cos^2 \psi)} \sin^2 \psi \left(d\chi + \frac{2Ma}{Q^2 + 2a^2}(\rho - 1)d\tau \right)^2. \end{aligned} \quad (3.7)$$

Finally after euclidean continuation and another change of variables,

$$\tau = -i\theta, \quad \rho = \cosh \eta, \quad (3.8)$$

with θ interpreted as a periodic coordinate with period 2π , we get

$$\begin{aligned} ds^2 & \equiv \bar{g}_{\mu\nu} dx^\mu dx^\nu \\ & = (Q^2 + a^2 + a^2 \cos^2 \psi) (d\eta^2 + \sinh^2 \eta d\theta^2 + d\psi^2) \\ & \quad + \frac{(Q^2 + 2a^2)^2}{(Q^2 + a^2 + a^2 \cos^2 \psi)} \sin^2 \psi \left(d\chi - i \frac{2Ma}{Q^2 + 2a^2} (\cosh \eta - 1) d\theta \right)^2. \end{aligned} \quad (3.9)$$

Let us now consider the limit in which the charge and the angular momenta become large as

$$q \sim \Lambda, \quad J \sim \Lambda^2, \quad (3.10)$$

for some large number Λ . In this limit $Q, a \sim \Lambda$ and the horizon area A_H scales as Λ^2 . It follows from the analysis of [9] that in this limit the one loop quantum correction to the entropy has a term proportional to $\ln \Lambda$, given by

$$\ln \Lambda \left[\sum_r (\beta_r - 1) N_r - 2\pi \int_{\text{horizon}} d\psi d\phi G(\psi) a_4(x) \right], \quad (3.11)$$

where

$$G(\psi) = \sqrt{\det \bar{g}} / \sinh \eta = (Q^2 + a^2 + a^2 \cos^2 \psi)(Q^2 + 2a^2) \sin \psi, \quad (3.12)$$

and $\sum_r (\beta_r - 1) N_r$ is the zero mode contribution evaluated as follows. The sum over r represents the sum over various fields, – in this case the metric and the U(1) gauge field. β_r 's are constants defined so that the Jacobian of changing variables from integration over the fields to integration over the zero mode deformation parameters gives a factor of Λ^{β_r} per zero mode. In particular $\beta_r = 1$ for the four dimensional gauge fields and 2 for the four dimensional metric [9]. N_r represents the regularized number of zero modes for each field. Computation reviewed in [9] shows that $N_r = -1$ for each four dimensional gauge field and $N_r = -3 - K$ for the four dimensional metric where K denotes the number of rotational isometries of the black hole solution. The -3 in the latter expression comes from the zero modes of the metric on AdS_2 whereas the $-K$ factor comes from the K gauge fields on AdS_2 coming from the dimensional reduction of the metric along the angular coordinates.² Thus for example for the Kerr-Newmann black hole we have $K = 1$ and hence

$$\sum_r (\beta_r - 1) N_r = (1 - 1) \times (-1) + (2 - 1) \times (-3 - 1) = -4. \quad (3.13)$$

The non-zero mode contributions to (3.11) have been discussed earlier, *e.g.* in [5, 12] but without taking into account the effect of the background gauge fields.

Now for the solution (3.1) we have [27, 28]

$$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = \frac{8}{(r^2 + a^2 \cos^2 \psi)^6} \left\{ 6M^2(r^6 - 15a^2 r^4 \cos^2 \psi + 15a^4 r^2 \cos^4 \psi - a^6 \cos^6 \psi) \right. \\ \left. - 12MQ^2 r(r^4 - 10r^2 a^2 \cos^2 \psi + 5a^4 \cos^4 \psi) \right\}$$

²It may appear strange that the number of zero modes is negative, but the actual number of zero modes is given by $(1 - \cosh \eta_0)$ times the number quoted here. Here η_0 is an infrared cut-off on the coordinate η . Thus for example the number of zero modes of the metric on AdS_2 is given by $3(\cosh \eta_0 - 1)$ which is clearly a positive number. The contribution proportional to $\cosh \eta_0$ can however be cancelled by boundary counterterms (or equivalently absorbed into a redefinition of the ground state energy) and only the term proportional to -3 contributes to the entropy.

$$\begin{aligned}
& +Q^4(7r^4 - 34r^2a^2 \cos^2 \psi + 7a^4 \cos^4 \psi) \Big\} \\
R_{\mu\nu}R^{\mu\nu} &= \frac{4Q^4}{(r^2 + a^2 \cos^2 \psi)^4}.
\end{aligned} \tag{3.14}$$

In the extremal limit we get

$$\begin{aligned}
\int_{\text{horizon}} d\psi d\phi G(\psi) R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} &= \frac{8\pi}{b(b^2 + 1)^{5/2}(2b^2 + 1)} \\
&\quad \left\{ 3(2b^2 + 1)^2 \tan^{-1} \left(\frac{b}{\sqrt{b^2 + 1}} \right) \right. \\
&\quad \left. + b\sqrt{b^2 + 1}(-8b^6 - 20b^4 - 8b^2 + 1) \right\}, \\
\int_{\text{horizon}} d\psi d\phi G(\psi) R_{\mu\nu} R^{\mu\nu} &= \frac{2\pi}{b(b^2 + 1)^{5/2}(2b^2 + 1)} \\
&\quad \left\{ 3(2b^2 + 1)^2 \tan^{-1} \left(\frac{b}{\sqrt{b^2 + 1}} \right) \right. \\
&\quad \left. + b\sqrt{b^2 + 1}(8b^2 + 5) \right\},
\end{aligned} \tag{3.15}$$

where

$$b = a/Q. \tag{3.16}$$

Note that b remains fixed under the scaling (3.10). Using (2.17), (3.11), (3.13), (3.15) and the result $A_H \sim \Lambda^2$ we now get the logarithmic correction to black hole entropy to be

$$\begin{aligned}
-2 \ln A_H - \frac{1}{720} \ln A_H \frac{1}{b(b^2 + 1)^{5/2}(2b^2 + 1)} &\left\{ 1233(2b^2 + 1)^2 \tan^{-1} \left(\frac{b}{\sqrt{b^2 + 1}} \right) \right. \\
&\quad \left. - b\sqrt{b^2 + 1}(-463 + 3080b^2 + 7960b^4 + 3184b^6) \right\}.
\end{aligned} \tag{3.17}$$

This describes logarithmic correction in the microcanonical ensemble where J_3 is fixed to J but \vec{J}^2 is arbitrary. As discussed in [9], if we also fix \vec{J}^2 to $J(J+1)$ then in the scaling limit (3.10) the coefficient of the $\ln A_H$ term does not change. On the other hand if we consider the Reissner-Nordstrom black hole and fix both J_3 and \vec{J}^2 to 0, then the number K of rotational isometries changes from 1 to 3, and as a result the zero mode contribution changes from $-2 \ln A_H$ to $-3 \ln A_H$.

We can get the result for Kerr black hole by taking the $b \rightarrow \infty$ limit of (3.17). This gives

$$-2 \ln A_H + \frac{199}{90} \ln A_H, \tag{3.18}$$

in agreement with the result of [9] (eq.(3.12) with $n_V = 1$, $n_S = n_F = n_{3/2} = 0$). On the other hand to get the result for the extremal Reissner-Nordstrom black hole with $\vec{J} = 0$, $\vec{J}^2 = 0$ we need to take the $b \rightarrow 0$ limit, and replace the zero mode contribution $-2 \ln A_H$ by $-3 \ln A_H$ since the number of rotational isometries now goes up to 3. This gives

$$-3 \ln A_H - \frac{106}{45} \ln A_H. \quad (3.19)$$

This agrees with the result of [8] (eq.(4.40)).

4 Discussion

As mentioned earlier, our result (3.17) puts strong constraint on any attempt at a microscopic explanation of the entropy of extremal Kerr-Newmann black holes. Such a theory must reproduce not only the leading Bekenstein-Hawking term but also the subleading logarithmic correction given in (3.17). In particular if there is an underlying two dimensional conformal field theory behind the entropy of the extremal Kerr-Newmann black hole, as has been suggested in [15], then (3.17) could be used as a guideline to search for this underlying CFT. In this context we would like to note that (3.17) is different from the universal form of the logarithmic correction that appears in the Cardy limit. This is not necessarily a contradiction to the Kerr/CFT hypothesis since the limit involved here corresponds to taking the central charge as well as the L_0 eigenvalue to be large and of the same order [15] whereas the Cardy limit corresponds to taking the L_0 eigenvalue to be large keeping the central charge fixed. Indeed, even for supersymmetric extremal black holes for which the macroscopic and the microscopic computations of the logarithmic correction to the entropy match, both the macroscopic and the microscopic results differ from what one would expect by a naive application of the Cardy formula [9].

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